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$\mathcal{O}(\alpha_S^2)$ Heavy Flavoured Jets in e^+e^- Collisions

with C. Oleari

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- Comment on DELPHI's study on the b mass from shape variables

Motivations

Although mass effects are bound to be not very important at LEP, there are quantities, like the momentum correlation of the quark–antiquark pair, which cannot be computed with the available (massless) calculations.

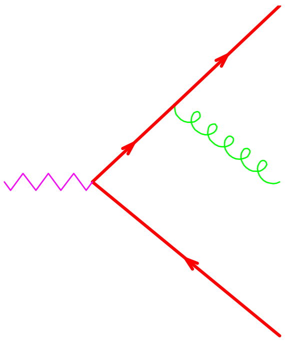
The initial motivation for this work has been to better understand dynamical correlations in heavy quark production, because of its impact upon the determination of R_b.

Other possible applications of these result are in the area of fragmentation function studies, QCD studies (effect of a finite m_b on the determination of α_S from jets), and, eventually, study of $t\bar{t}$ production at an NLC.

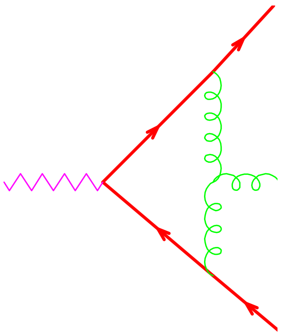
Calculation

When considering only unoriented quantities, the ingredients for the computation of heavy flavour production at $\mathcal{O}(\alpha_S^2)$ are the following

- 0-loop and 1-loop graphs for the production of the $q\overline{q}$ pair plus a gluon



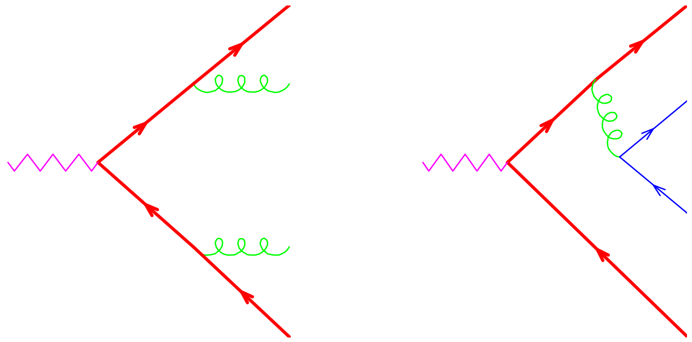
$0-$loop



$1-$loop

The square and interference of 0-loop graphs among themselves yields $\mathcal{O}(\alpha_S)$ terms, while the interference of 0-loop graphs with 1-loop graphs yields $\mathcal{O}(\alpha_S^2)$ terms.

- 0-loop graphs for the production of the $q\overline{q}$ pair plus two more partons

  The square and interferences of the above graphs among themselves yield $\mathcal{O}(\alpha_S^2)$ effects.

- The total cross section for the production of a $b\overline{b}$ pair at order $\mathcal{O}(\alpha_S^2)$. This is known exactly up to $\mathcal{O}(\alpha_S)$. The $\mathcal{O}(\alpha_S^2)$ terms are not known exactly; however, good approximations exist when $m_b/s \ll 1$ (K.G. Chetyrkin, R. Harlander, J.H. Kühn and M. Steinhauser)

It is obvious that, given the first two ingredients, the only missing piece is the term with a single $q\overline{q}$ pair in the final state. If we only consider unoriented quantities, the kinematics of this state is fully specified. Thus it is enough to know the total cross section to get it.

Results

Several groups have performed the calculation:
C. Oleari, P.N.
G. Rodrigo
W. Bernreuther, A. Brandenburg and P. Uwer
Comparisons are under way...

At LEP, much work on jet shape variables has been done using the program EVENT, developed for the LEP1 Yellow Book (Kunszt, P.N.). The program provides a way to compute the $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ coefficients for any jet quantity. The result of the present calculation is also implemented in a program of this kind, which we call a parton Montecarlo generator. The program outputs correlated pairs of 3-parton and 4-parton events with a weight. Shape variables are computed and histogrammed for these partons. The program can yield infinite answers, if the shape variable one is considering are not infrared finite, as well as negative answers, since it is a fixed order program, and there are kinematic regions where radiative corrections are large, and need to be resummed to yield a sensible result.

A simple example of how this work: suppose we have a process with a final state characterized by a variable $0 < x < 1$, where $x = 1$ at the Born level, and

$$\frac{d\sigma}{dx} = \delta(1-x) + \alpha\frac{1}{1-x+\eta} - \alpha\log\frac{1}{\eta}\delta(1-x) + \mathcal{O}(\alpha_S^2)$$

The second term is the real production term, the last one is the virtual term, whose final state is the same as the Born term, and η is a small cutoff, that regulates the infrared divergences. It is easy to see that, if we compute a quantity $G(x)$ with this formula, we get

$$\langle G(x)\rangle = G(1) + \alpha\int_0^1 dx\frac{G(x)-G(1)}{1-x+\eta}$$

which has a finite limit as $\eta\to 0$ provided G is continuous for $x = 1$ (e.g. infrared safe).

A parton Montecarlo generator that implements the above formula is made as follows

- Generate 1 event with weight 1 and $x = 1$
- Generate N random $\tilde{x}$ (with $0 < \tilde{x} < 1$); from each $\tilde{x}$ generate two events, one with $x = \tilde{x}$ and weight $1/N\,\alpha\frac{1}{1-\tilde{x}+\eta}$ and the other with $x = 1$ and weight $-1/N\,\alpha\frac{1}{1-\tilde{x}+\eta}$.

It is easy to see that, if we compute a quantity $G(x)$ with this Montecarlo we get

$$\langle G(x)\rangle = G(1) + \alpha\int_0^1 dx\,\frac{G(x)-G(1)}{1-x+\eta}$$

Provided G is infrared safe, we can keep $\eta = 0$ directly, and get a finite answer.

Experience with jets at LEP1 has proven that these programs are very valuable to experimentalists, since they can compute any kind of shape variables, and especially modify them to adapt to the experimental configurations.

Applications

Applications of this calculation cover the following areas

- Correlations in $b\bar{b}$ momenta (Oleari, P.N.): small, $R_{\rm b}$ is safe from this point of view (not very fashionable any more).

- Fragmentation function studies at fixed order (Oleari, P.N.). Can compute $D_{b/c}E(z)$ at order $\mathcal{O}(\alpha_S^2)$. The standard evolution calculation gives only terms of the order $1+\mathcal{O}(\alpha_S)+\mathcal{O}(\alpha_S^2\log Q/m)$, the present calculation allows one to get also the $\mathcal{O}(\alpha_S^2\log Q/m)$ term with no logarithmic enhancement.

- Jet studies, flavour independence of strong charge, b mass measurement, G. Rodrigo, A. Santamaria and M. Bilenkii, Juan Fuster (DELPHI).

- Future: top production.

The running of the b mass

We use the term mass with two meanings:

1. The rest energy of a particle, m_{p}.

2. A parameter in the Lagrangian, $\overline{m}$.

The meaning (1) for a heavy quark has an intrinsic uncertainty, due to the fact that quarks are confined in hadrons (of size $\approx 1/300$ MeV^{-1}), and therefore have momenta of the order of ≈ 300 MeV. It is however well defined for very heavy quarks.
The meaning (2) is related to (1) in an indirect way. We can use (2) to compute (1), and then

$$m_{\mathrm{p}} = m_{\mathrm{p}}(\overline{m}, \alpha_S, \mu)$$

where μ is a renormalization scale. We can think of it as of an ultraviolet cutoff: energies much higher then μ decouple from the dynamics. We can change the value of μ, α_S, and $\overline{m}$, keeping the physics unchanged. In other words,

$$m_{\mathrm{p}} = m_{\mathrm{p}}(\overline{m}(\mu), \alpha_S(\mu), \mu)$$

does not depend upon μ. Thus, if we measure m_{p} we measure also $\overline{m}$, if we know α_S. What does it mean to say that one observes the running of $\overline{m}$?

The only reasonable answer is the following: there are physical quantities which are expressed more naturally as a functions of $\overline{m}$. One example: the axial vector part of Z decay into $b\bar{b}$

$$\Gamma_b^{(\mathrm{A})} = \frac{g_{\mathrm{A}}^2 g^2 m_{\mathrm{Z}}}{\cos^2\theta_{\mathrm{W}} 64\pi} \left(1 - 6r_b + \frac{\alpha_S}{\pi}\left[1 - 6r_b(2\,\log r_b + 1)\right]\right)$$

where $r_b = m_{\mathrm{P}}^2/m_{\mathrm{Z}}$. Since

$$m_{\mathrm{P}}^2 = \overline{m}^2(m_{\mathrm{Z}}^2)\left[1 + 2\frac{\alpha_S}{\pi}\left(\frac{4}{3} - \log r_b\right)\right]$$

we can also write ($\bar{r}_b = \overline{m}^2(m_{\mathrm{Z}})/m_{\mathrm{Z}}^2$)

$$\Gamma_b^{(\mathrm{A})} = \frac{g_{\mathrm{A}}^2 g^2 m_{\mathrm{Z}}}{\cos^2\theta_{\mathrm{W}} 64\pi} \left(1 - 6\bar{r}_b + \frac{\alpha_S}{\pi}\left[1 - 22\bar{r}_b\right]\right)$$

more natural: no large $\log r_b$ coefficients in α_S term! There are deep reasons why, for this quantity, the b mass acts more like a coupling constant, rather then a rest energy. Total width are related to vacuum polarization at short distances, and these last quantities couldn't care less about the heavy quark rest energy. Another well known example is the Higgs width into $b\bar{b}$. There, one measures directly the Higgs coupling, which generates the mass of the b via symmetry breaking in the Standard Model.

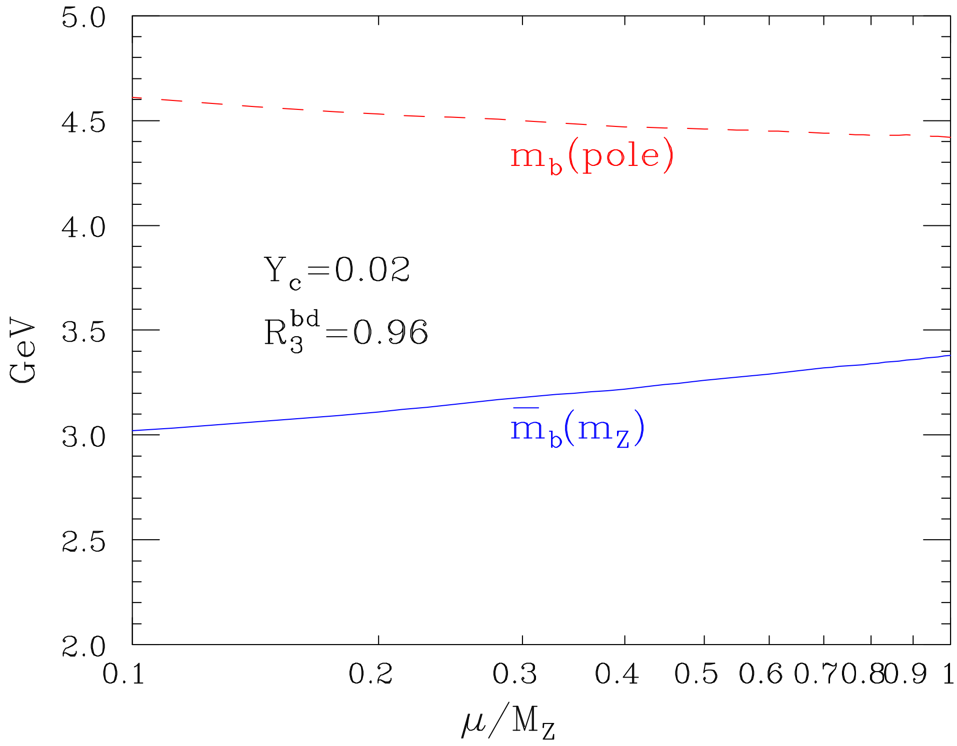
Does something similar happen in the jet cluster multiplicity used by DELPHI?
Look at Rodrigo, Santamaria, Bilenky

We have

$$R_3^{bd} - 1 = r_b \left[-18.62 + \frac{\alpha_S}{\pi} \left(115.36 + 9 \log r_b\right) \right]$$

$$R_3^{bd} - 1 = \overline{r}_b \left[-18.62 + \frac{\alpha_S}{\pi} \left(65.71 + 46.24 \log \overline{r}_b\right) \right]$$

Which one is more natural? Can use one or the other, but certainly cannot claim that we have observed the running.



Thus:

still very interesting, but running is much more difficult!