$\mathrm{B\bar{B}}$ mixing from $\mathrm{e^+e^-}$ colliders

EPS-HEP Brussels

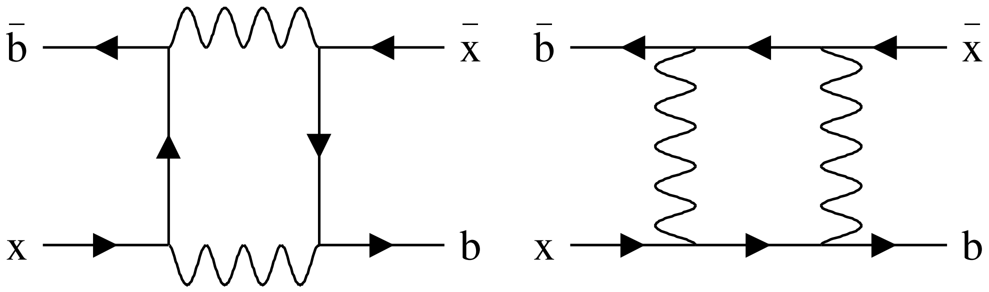
27 July 1995

P. S. Wells

CERN, Geneva, Switzerland.

- Introduction to flavour mixing in the B sector
- Measurements at the $\Upsilon(4S)$
- Measurements of Δm_{d} at LEP
- Implications for fraction of $\mathrm{B_s^0}$ in $\mathrm{Z} \to \mathrm{b\overline{b}}$ events, f_{s}
- Limits on Δm_{s} from LEP
- Implications for $|V_{\mathrm{ts}}/V_{\mathrm{td}}|$

$\mathrm{B}\overline{\mathrm{B}}$ mixing in the Standard Model



Calculate the mass difference between B^0 and $\overline{\mathrm{B}^0}$:

$$\Delta m_{\mathrm{x}} = \frac{G_{\mathrm{F}}^2 m_{\mathrm{W}}^2}{6\pi^2} y_{\mathrm{t}} f(y_{\mathrm{t}}) \eta_{\mathrm{B}} m_{\mathrm{B}} B_{\mathrm{B}} f_{\mathrm{B}}^2 |V_{\mathrm{tx}}^* V_{\mathrm{tb}}|^2$$

- $\mathrm{x} \equiv \mathrm{d}$ or s
- $y_{\mathrm{t}} \equiv m_{\mathrm{top}}^2 / m_{\mathrm{W}}^2$
- η_{B}, m_{B}, B_{B} and f_{B}^2 are the QCD correction, mass, bag constant and decay constant for the B_{x} meson

Many uncertainties cancel in the ratio:

$$\frac{\Delta m_{\mathrm{s}}}{\Delta m_{\mathrm{d}}} = \frac{m_{\mathrm{B}_s}}{m_{\mathrm{B}_d}} \frac{B_{\mathrm{B}_s} f_{\mathrm{B}_s}^2}{B_{\mathrm{B}_d} f_{\mathrm{B}_d}^2} \left| \frac{V_{\mathrm{ts}}}{V_{\mathrm{td}}} \right|^2$$

($\sqrt{B_{\mathrm{B}_s} f_{\mathrm{B}_s}^2 / B_{\mathrm{B}_d} f_{\mathrm{B}_d}^2} = 1.16 \pm 0.1$ Ref. Ali + London).

Mixing observables

Neglecting CP violation, mass eigenstates are:

$$\mathrm{B}_1 = \frac{1}{\sqrt{2}}(\mathrm{B}^0 + \overline{\mathrm{B}^0}), \quad \mathrm{B}_2 = \frac{1}{\sqrt{2}}(\mathrm{B}^0 - \overline{\mathrm{B}^0}).$$

If B^0 produced at $t = 0$,

$$\begin{aligned} \mathcal{P}(\mathrm{B}^0, t) &= \frac{1}{2\tau_\mathrm{B}} e^{-t/\tau_\mathrm{B}} (1 + \cos \Delta m t), \\ \mathcal{P}(\overline{\mathrm{B}^0}, t) &= \frac{1}{2\tau_\mathrm{B}} e^{-t/\tau_\mathrm{B}} (1 - \cos \Delta m t), \end{aligned}$$

Define $x \equiv \tau_\mathrm{B}\Delta m$ (i.e. $2\pi\times$ no. oscillations per τ_B).

First observations were of time integrated probability:

$$\chi \equiv \mathcal{P}(\mathrm{B}^0 \text{ decays as } \overline{\mathrm{B}^0}) = \frac{1}{2}\frac{x^2}{1+x^2}.$$

At LEP (or in $\mathrm{p}\overline{\mathrm{p}}$), observable is $\chi = f_\mathrm{d}\chi_\mathrm{d} + f_\mathrm{s}\chi_\mathrm{s}$.

At $\Upsilon(4S)$ only B^0_d produced – measure χ_d from fraction of mixed events.

LEP (and CDF) have observed time dependence for B^0_d, and measured Δm_d directly.

Mixing at the $\Upsilon(4S)$

$\mathrm{B}^0\overline{\mathrm{B}^0}$ produced nearly at rest and in a coherent state.

Fraction of mixed events is related to χ_d by:

$$\chi_\mathrm{d} = \frac{N(\mathrm{B}^0\mathrm{B}^0) + N(\overline{\mathrm{B}^0\mathrm{B}^0})}{N(\mathrm{B}^0\mathrm{B}^0) + N(\overline{\mathrm{B}^0\mathrm{B}^0}) + N(\mathrm{B}^0\overline{\mathrm{B}^0})}$$

Experimentally, need to identify the flavour of both B's in the event, and account for backgrounds:

- Fully reconstructed events (Original unambiguous observation of mixing, low statistics)
- $\ell^\pm\ell^\pm$: Fraction R of lepton pairs with same charge gives $\chi_\mathrm{d} = (1+\lambda)R$
  Correct for $\mathrm{B}^+\mathrm{B}^-$ via $\lambda = f_+b_+^2/f_0b_0^2$
  (f meson fraction, b semilept. BR)
- $\mathrm{B}_\mathrm{d}^0 \rightarrow \mathrm{D}^{*-}\ell^+\nu$ with $\mathrm{D}^* \rightarrow \mathrm{D}^0\pi_\mathrm{s}$. Partial reconstruction techniques used to identify this decay. Flavour of from other B from ℓ.

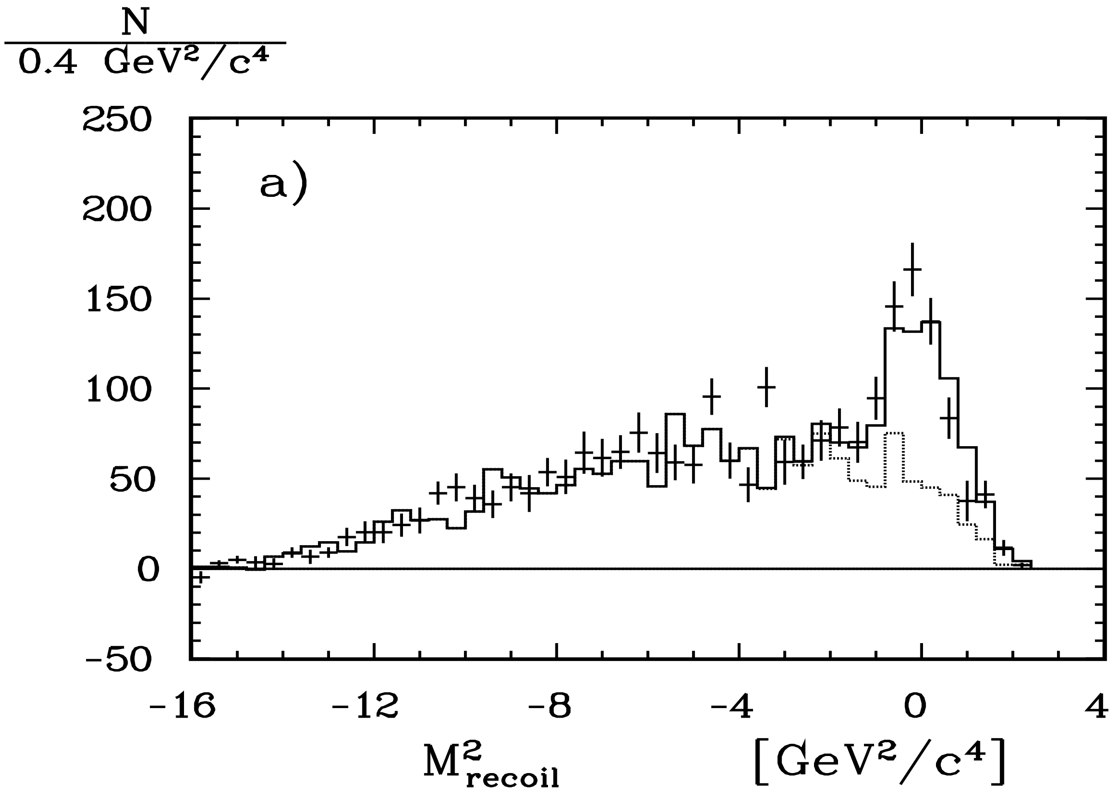
New result from ARGUS

Method 1 $\overline{\mathrm{B_d}^0} \longrightarrow \mathrm{D}^{*+}X$. Fully reconstruct:

$\mathrm{D}^{*+} \longrightarrow \mathrm{D}^0{\pi_s}^+,\ \mathrm{D}^0 \longrightarrow \mathrm{K}^-\pi^+$ or $\mathrm{D}^0 \longrightarrow \mathrm{K}^-\pi^+\pi^+\pi^-$

Method 2 $\mathrm{B_d^0} \longrightarrow \mathrm{D}^{*-}\ell^+\nu$ (E, p of D^* inferred from π_s)

$$M^2_{\mathrm{recoil}} = (E_{\mathrm{beam}} - E_{\mathrm{D}^*} - E_\ell)^2 - (p_{\mathrm{D}^*} + p_\ell)^2$$

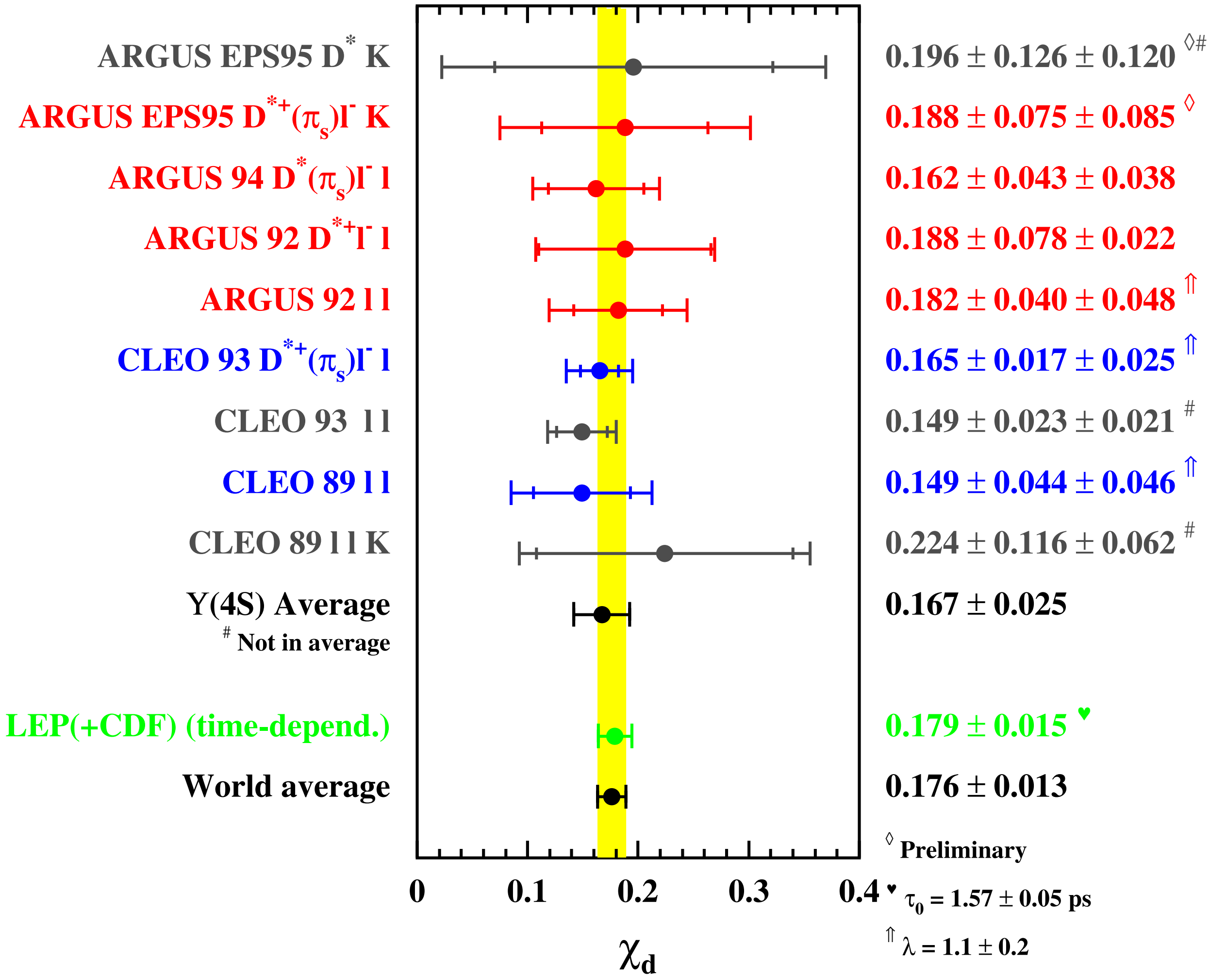


New feature for both methods: flavour of other B from additional $\mathrm{K}^\pm$

$$N(\mathrm{D}^{*+}\mathrm{K}^-)/N(\mathrm{D}^{*+}\mathrm{K}^+) = f(\chi_{\mathrm{d}})$$

where complete expression depends on BR's.

Average value of χ_d



- Year refers to publication (or conference).
- Less precise results with unknown statistical overlap excluded from average
- Common systematic errors taken into account: value of $\lambda = 1.1 \pm 0.2$ and charm fraction in lepton samples
- Latest $\tau(\mathrm{B}^0)$ used to convert Δm_d(LEP+CDF) into χ_d (Ref. talk by H.–G. Moser)

Measurements of Δm_{d} at LEP

General considerations:

- Select an enriched B^0 sample
- Identify B^0 or $\overline{\mathrm{B}^0}$ flavour at production and decay
- Measure the B^0 decay length, L, and the boost to estimate the proper time $t = L/\beta\gamma c = L m_{\mathrm{B}}/p_{\mathrm{B}} c$
- Fit the number of mixed and unmixed events as a function of t, with Δm_{d} as a free parameter, taking into account all contributions to the sample
- May fit as a function of L rather than t
- Possibly binned fit to $R = N_{\mathrm{mix}}/(N_{\mathrm{mix}} + N_{\mathrm{unmix}})$ or charge correlation $Q = (N_{\mathrm{unmix}} - N_{\mathrm{mix}})/(N_{\mathrm{unmix}} + N_{\mathrm{mix}})$
- May allow extra parameters in the fit, eg. Δm_{s}, charge tagging efficiency, background fractions

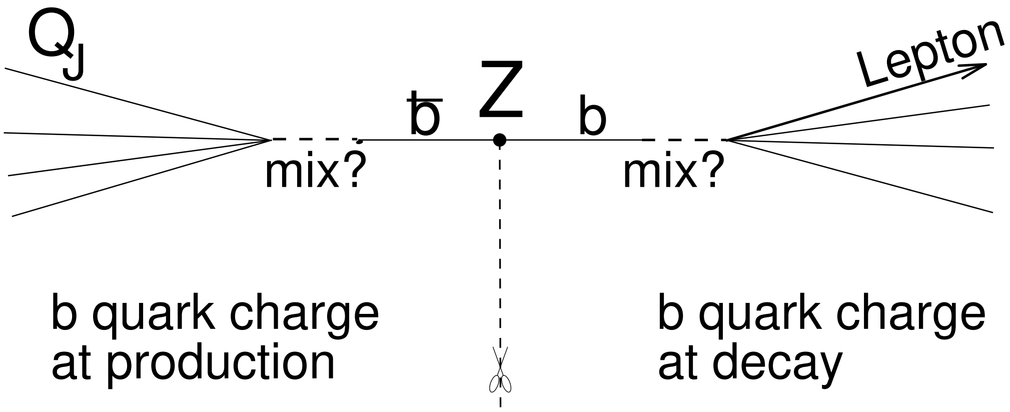
Analyses using $\ell\ell$ or ℓQ_{J}

- Sensitive to Δm_{d} and Δm_{s}

Analyses using $\mathrm{D}^{*+}(\ell^-)$ opposite ℓ or Q_{J}

- Relatively pure $\mathrm{B}^0_{\mathrm{d}}$ sample

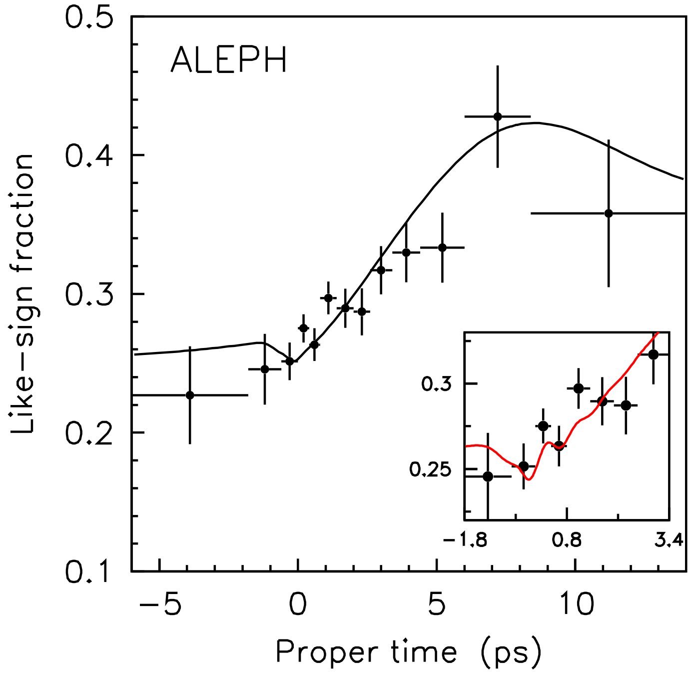
Measurements using $\ell\ell$ or ℓQ_J



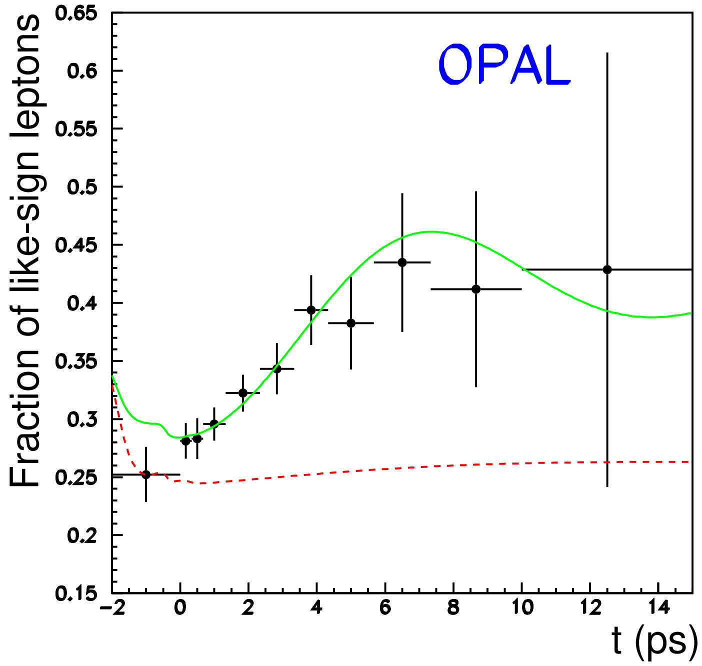
- Jet (or hemisphere) charge* : $Q_\mathrm{J} = \dfrac{\sum_i q_i (p_i^{||})^\kappa}{\sum_i (p_i^{||})^\kappa}$
- Measure L from secondary vertex including ℓ and generic D decay products. Typical resolution 350μm
- Boost estimate from charged tracks at vertex, neutral energy in jet, plus missing energy (ALEPH, DELPHI) or two body kinematics (OPAL). Typical resolution 15%
- Fit like– and opposite–sign $\ell\ell$ or product $Q_\ell \times Q_\mathrm{J}$ vs. proper time for all ℓ vertices
- Fitting procedure NOT trivial – contributions from $\mathrm{B_d^0}$, $\mathrm{B^+}$, $\mathrm{B_s^0}$ and Λ_b, $\mathrm{b} \to \mathrm{c} \to \ell$, $\mathrm{c\bar{c}}$, fakes
- Dominant systematic errors: B lifetimes, f_d, f_s, non-$\mathrm{b} \to \ell$ background, time resolution

*(Actual definition may vary)

Like-sign fraction vs. proper time



Fit shown with maximal $\mathrm{B_s^0}$ mixing, zoom shows Δm_s free



Fitted curves: $\Delta m_\mathrm{s} = 10\mathrm{ps}^{-1}$. Dashed curve: no $\mathrm{B_d^0}$ mixing.

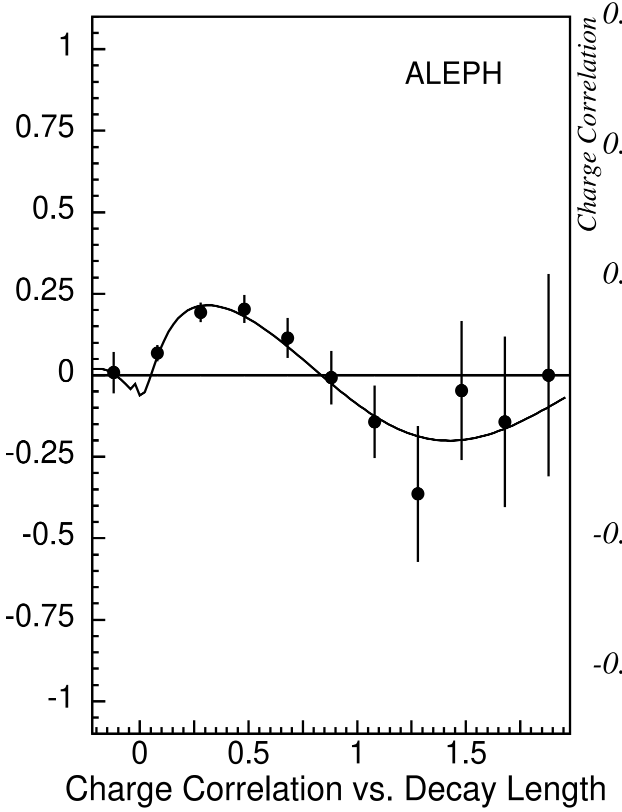
Measurements using D^* mesons

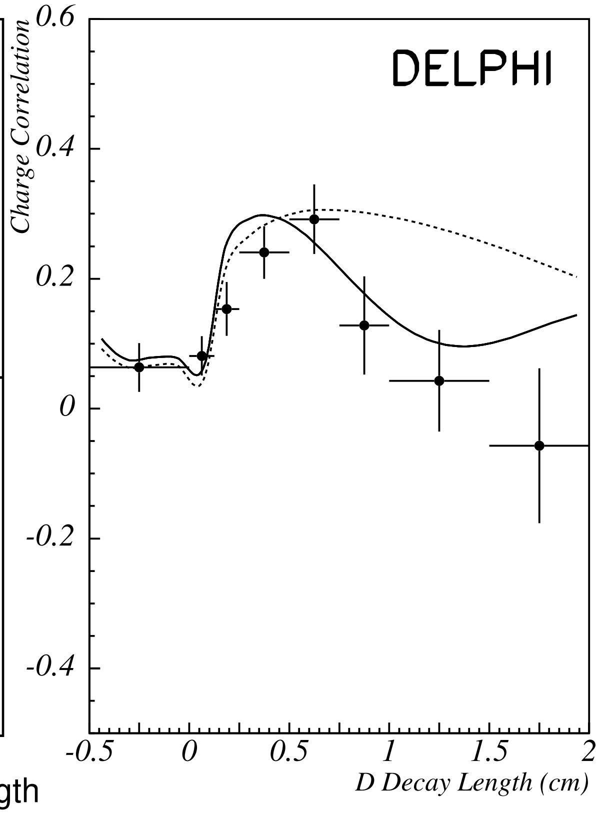
- $\mathrm{B_d^0}$ rich sample with a D^{*+} tag
- D^{*+} automatically gives decay flavour
- $\mathrm{D}^{*+} \rightarrow \mathrm{D}^0 \pi_\mathrm{s}{}^+$, with
  $\mathrm{D}^0 \rightarrow \mathrm{K}^-\pi^+$, $\mathrm{K}^-\pi^+\pi^0$ or $\mathrm{K}^-\pi^+\pi^+\pi^-$
- Improve $\mathrm{B_d^0}$ purity and L estimate by looking for $\mathrm{D}^{*+}\ell^-$ (allows reconstruction of $\mathrm{B_d^0}$ vertex)
- Identify production flavour from ℓ in opposite jet, or a jet charge measure using both the $\mathrm{B_d^0}$ jet and opposite jet* :
  $Q_{\mathrm{2jet}} = Q_\mathrm{J}{}^{\kappa=0}(\mathrm{B_d^0}\text{ jet}) - 10 Q_\mathrm{J}{}^{\kappa=1}(\text{opp jet})$
- Fit for mixed fraction or charge correlation as a function of decay length or proper time
- Dominant systematics from B^+ fraction and mistag rate in sample
- May allow mistag rate and/or fractions to be free in the fit in addition to Δm_d

*(Actual definition may vary)

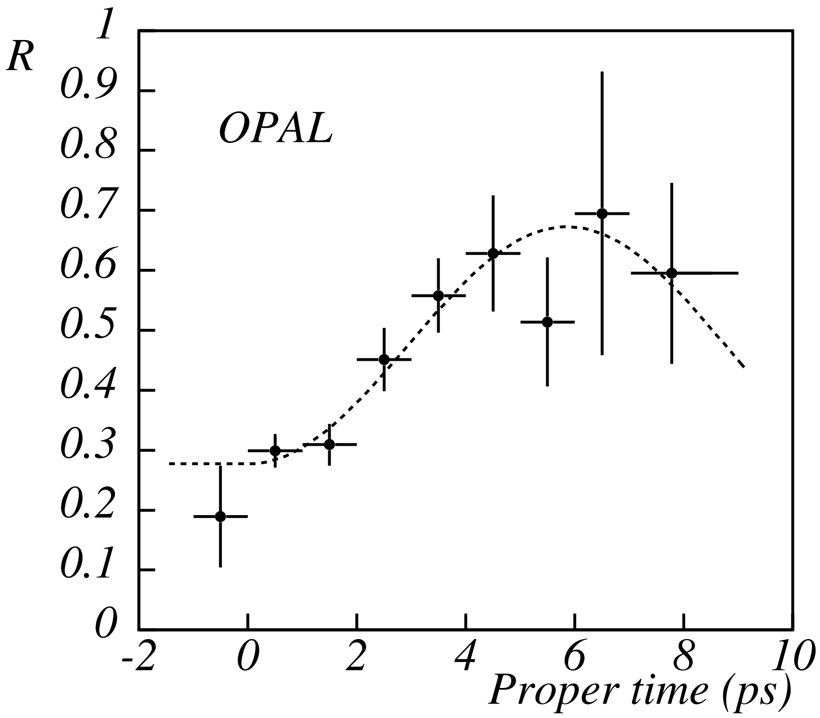
Fit results with D^* mesons

$(N_{\mathrm{unmix}} - N_{\mathrm{mix}})/(N_{\mathrm{unmix}} + N_{\mathrm{mix}})$ vs. $L(\mathrm{D}^0)$

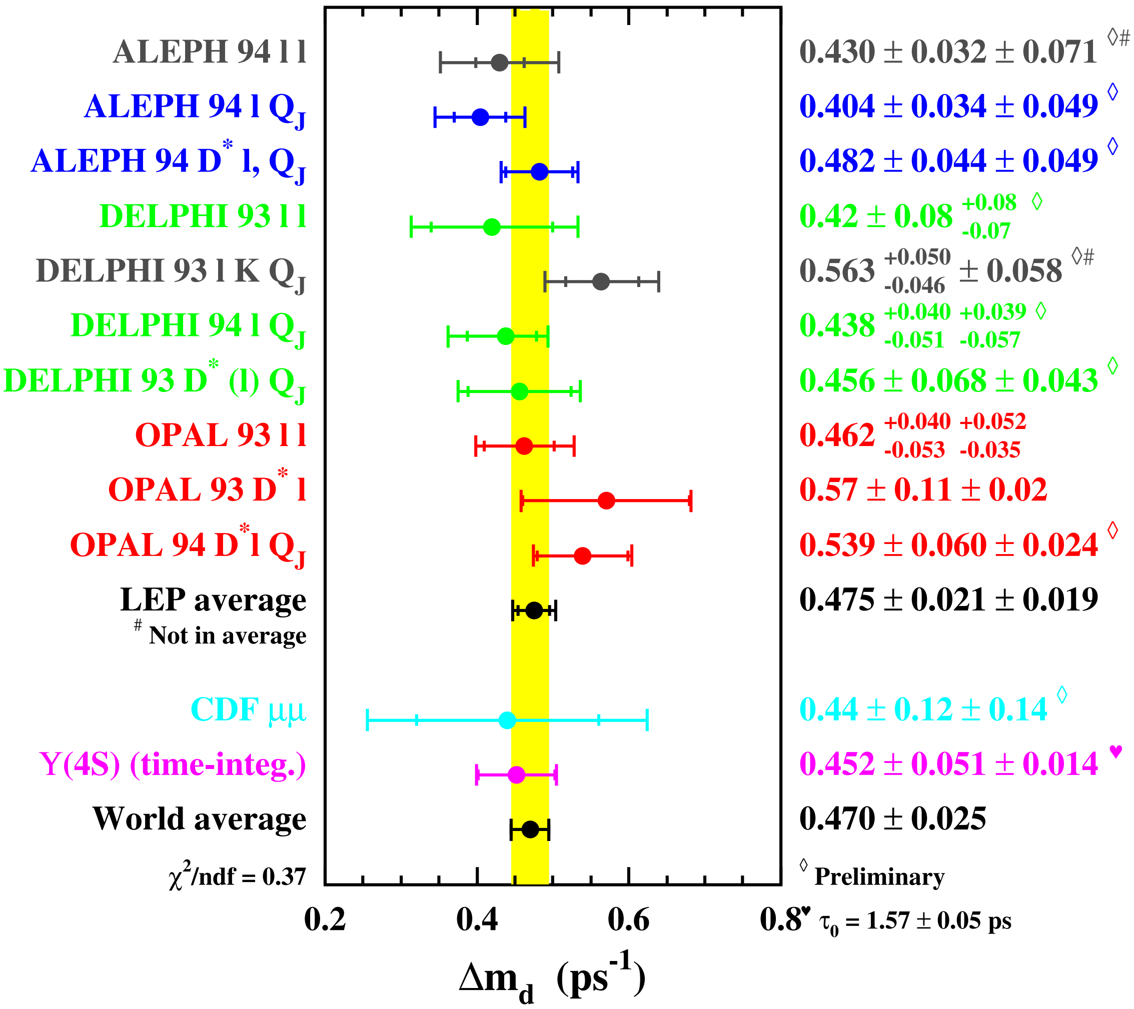




$R = N_{\mathrm{mix}}/(N_{\mathrm{mix}} + N_{\mathrm{unmix}})$ vs. B decay proper time



Δm_d Results



- Year refers to data collection.
- Less precise results with unknown statistical overlap excluded from average.
- Common systematic errors taken into account. Typical correlation coefficients are $\approx 25\%$ between $\ell\ell$ or $\ell Q_{\rm J}$, others $< 10\%$.
- Latest $\tau(\mathrm{B}^0)$ used to convert $\chi_{\rm d}(\Upsilon(4S))$ into $\Delta m_{\rm d}$ (Ref. talk by H.–G. Moser).

Inferred value of f_{s} from mixing

The fraction of (ground state) $\mathrm{B_s^0}$ mesons in $\mathrm{Z} \to \mathrm{b\overline{b}}$ events is not well known. Recent observations of $\mathrm{B_s^{0**}} \to \mathrm{B^{(*)}K}$ affect prediction of f_{s}.

Most measurements of $\overline{\chi} = f_{\mathrm{s}}\chi_{\mathrm{s}} + f_{\mathrm{d}}\chi_{\mathrm{d}}$ are from $\ell\ell$ analyses. Excluding ℓQ_{J} measurements, (different flavour mixture), LEP average is (Ref. talk by P. Perret):

$$\overline{\chi} = 0.115 \pm 0.006$$

Taking fractions of ground state b hadrons:

$$f_{\mathrm{u}} + f_{\mathrm{d}} + f_{\mathrm{s}} + f_{\Lambda} = 1$$

assuming $f_{\mathrm{u}} = f_{\mathrm{d}}$ and $\chi_{\mathrm{s}} = 0.5$ then

$$f_{\mathrm{s}} = \frac{2\overline{\chi} - (1 - f_{\Lambda})\chi_{\mathrm{d}}}{1 - \chi_{\mathrm{d}}}$$

Take $\chi_{\mathrm{d}} = 0.176 \pm 0.013$ and $f_{\Lambda} = 13 \pm 4\%$

$$f_{\mathrm{s}} = 9.4 \pm 1.5 \pm 1.2 \pm 0.9 = 9.4 \pm 2.1\%$$

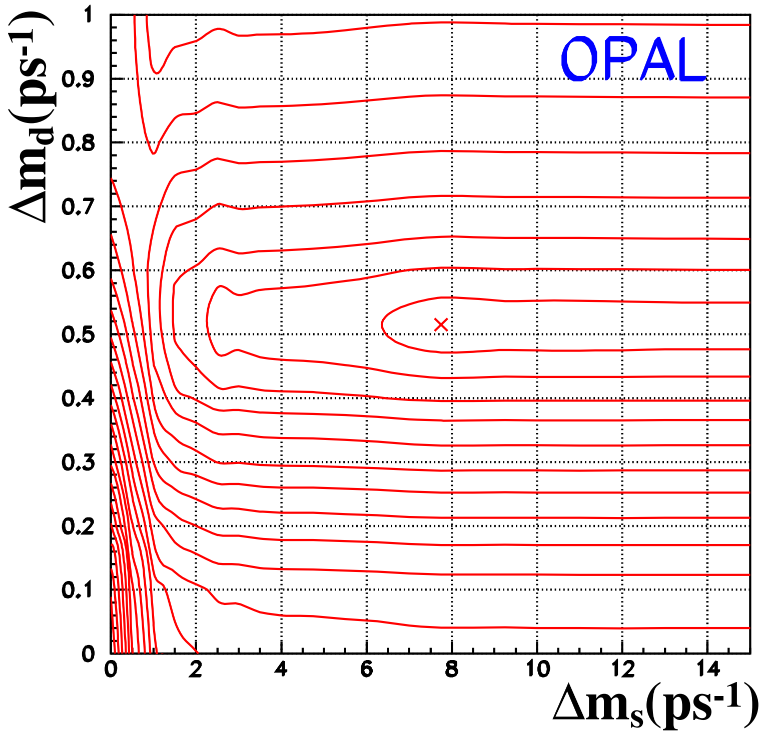
where separate errors are due to $\overline{\chi}$, χ_{d} and f_{Λ}.

Combining this with estimate from branching ratio measurements of $f_{\mathrm{s}} = 12 \pm 3\%$ gives

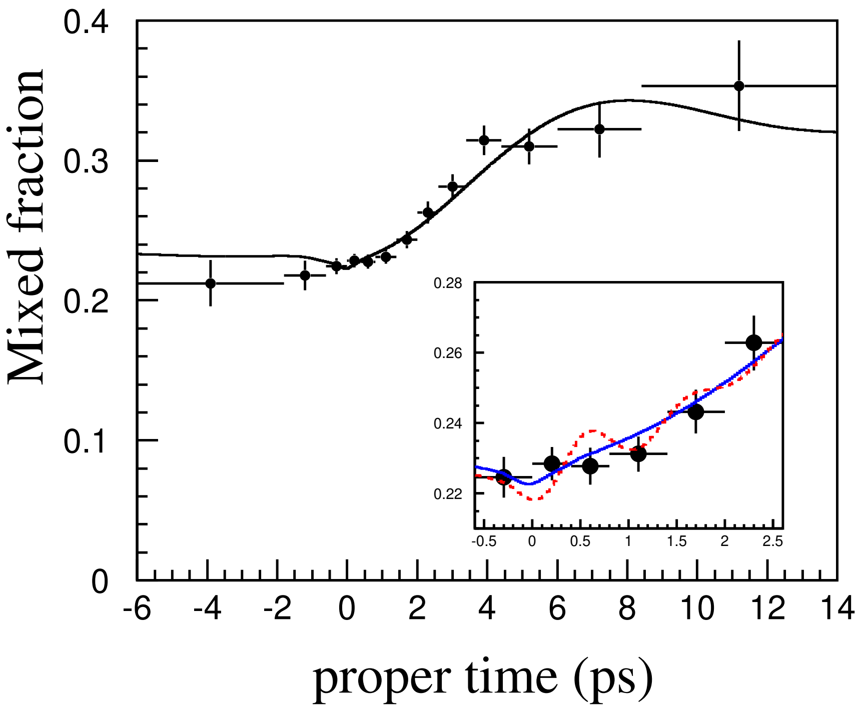
$$f_{\mathrm{s}} = 10 \pm 2\%$$

Investigation of Δm_s at LEP

OPAL $\ell\ell$: Log-likelihood contours in Δm_d–Δm_s plane.



ALEPH ℓQ_J: Zoom shows prediction for $\Delta m_s = 6\text{ps}^{-1}$



Limits on Δm_s at 95% C.L.

(Ref. talk by O. Schneider for details of limits)

		Δm_s ps^{-1}	f_s
ALEPH	$\ell K Q_J$	> 4.0	$12 \pm 3\%$
	$\ell\ell$	> 5.6	$12 \pm 3\%$
	ℓQ_J	> 6.1	12%
	ℓQ_J	> 5.6	10%
DELPHI	ℓQ_J	> 4.2	$10 \pm 3\%$
	$(D_s{}^+\ell^-)Q_J$	> 1.5	$76 \pm 10\%$
OPAL	$\ell\ell$	> 2.2	$12.0 \pm 3.6\%$

For $\Delta m_d = 0.470 \pm 0.025$ps^{-1} and Δm_s limit M_{95} calculate limits $\Delta m_s/\Delta m_d > M_{95}/0.495$ and $|V_{ts}/V_{td}| > \sqrt{1.27 M_{95}}$.

- Given f_s, choose $\Delta m_s > 5.6$ps^{-1} @ $95\% C.L.$
- Combining this Δm_s limit with measured Δm_d:

$$\frac{\Delta m_s}{\Delta m_d} > 11.3 \; @ \; 95\% C.L.$$

- The corresponding limit on CKM matrix elements is

$$\left|\frac{V_{ts}}{V_{td}}\right| > 2.7 \; @ \; 95\% C.L.$$

Summary

- World average $\chi_{\mathrm{d}} = 0.175 \pm 0.013$
- World average $\Delta m_{\mathrm{d}} = 0.470 \pm 0.025$
- These numbers are now dominated by direct observation of time dependent mixing
- Together with the average rate of mixing, they imply

$$f_{\mathrm{s}} = 9.4 \pm 2.1\%$$

- Take LEP limit as $\Delta m_{\mathrm{s}} > 5.6\mathrm{ps}^{-1} \,@\, 95\% C.L.$
- This implies

$$\left|\frac{V_{\mathrm{ts}}}{V_{\mathrm{td}}}\right| > 2.7 \,@\, 95\% C.L.$$

which is of similar precision to constraint from CKM unitarity of $|V_{\mathrm{ts}}/V_{\mathrm{td}}| > 2.8$ (Ref. Ali + London)